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Abstract

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We study the regime-dependent relationship between the performance of alternative beta strategies and the Fama-French factors. It is widely believed that the alphas generated by alternative beta strategies can be explained by their exposure to well-known pricing factors such as size and value. However, nothing is known about the dynamic of the relationship between the strategies and the factors in different market regimes. We estimate a four-regime Markov switching model on datasets including market portfolio, factor returns and returns of two alternative beta strategies - equal weight and minimum variance. Regime-switching regression of the strategies' returns conditional on four regimes shows that their factor exposures change significantly across regimes, notably in the case of the minimum variance strategy. This indicates that investment strategies adapt continuously to market conditions via periodic rebalancing and position their portfolios differently with respect to the factors when there is a regime change.

1 Introduction

The creation of an equal weighted version of the S&P 500 index and a respective ETF in 2003 paved the way for a trend of rethinking regarding traditional benchmarks and offering solutions that correct market portfolio inefficiencies¹. Ten years later, the family of so-called alternative beta indices has grown substantially, counting dozens of rule-based "new passive" approaches that tend to offer better risk-adjusted returns than their market capitalization counterparts². Two prominent members of this family are the already mentioned equal weight strategy that assigns the same weight to all constituents of the investment universe, and the minimum variance strategy that builds a fully invested portfolio of smaller possible variance via mathematical optimization. These two alternative beta solutions have quite different risk-returns profiles. The second is a defensive risk-concerned approach, while the first is diversification-oriented and emphasizes stocks outside familiar areas of the mega-cap sector.

The fast spread and popularity of alternative beta has led to a legitimate debate on the sources of outperformance of these strategies with respect to the market portfolio. For example, for both equal weight and minimum variance strategies, a significant alpha was documented in the literature, see Clarke et al. (2006) and DeMiguel et al. (2009). The classical linear regression approach using factor models attributed this alpha to well-known

asset pricing factors: in particular size and value (see Chow et al. (2011)). Other studies further claimed to explain the alternative beta outperformance only after enlarging the set of explaining factors with a beta anomaly factor and a residual risk factor (see Scherer (2010) and Carvalho et al. (2012))³. On the contrary, other authors argued that there are intrinsic features in the construction of the alternative beta strategies, as contrarian-like periodic rebalancing of equal weight portfolio, that explain a significant amount of outperformance and cannot be related to the factors (Plyakha et al. (2012)).

Though the idea of simplifying the picture by attributing the alternative beta outperformance to a linear mix of pricing anomaly factors is attractive, it gives only a partial answer for those who want to understand alternative beta strategies and use them in a portfolio. Equity markets have been shown to exhibit persistent regimes. Turner et al. (1989), Schaller and Van Norden (1997) and Maheu and McCurdy (2000) showed existence of distinct phases characterized by low and high volatility of the market. It was also shown that behavior of pricing anomaly factors change substantially in different regimes. Guidolin and Timmermann (2008) showed that the value and size factor returns during bear markets were not significantly different from zero and were positively correlated to the market portfolio. Instead, the factors had more complicated behavior in bull regimes. The value effect was generally strong but changed its correlation to the market from positive to negative in different bull regimes.

¹One can find a discussion of market inefficiencies in Fama and French (2004) and Markowitz (2005). For an overview of alternative beta strategies we refer the reader to Monnier and Rulik (2012).

²Though the alternative beta indices have relatively short lived history (for example, the first minimum variance index was launched in 2008 by MSCI), their back-tests are available for much more extended time periods, 10 years and more.

³The beta anomaly portfolio is a cash and beta-neutral portfolio of long low beta stocks and short high beta stocks, the residual risk anomaly is captured by a beta-neutral portfolio that is long the 20% of stocks with the lowest idiosyncratic risk and short the 20% stocks with the highest idiosyncratic risk

The size effect was negative in one bull regime (that included, in particular, the mid-1990s until the late 1990s bubble) while being positive in other bull regimes, and its correlation to the market switched sign as well. Long-term regressions used to relate the strategies to the factors do not account for this variability and give an averaged regime-insensitive result.

In this paper we take a conditional point of view and explore the relationship between the alternative strategies and the classical pricing factors, assuming that it can be different in different market regimes. It is logical to expect that alternative beta investment strategies, that adapt continuously to market conditions via periodic rebalancing, position their portfolios differently with respect to the pricing factors in different regimes. Existence of time variations in factor exposures has important implications for understanding the alternative beta strategies, and especially for implementation of a tactical allocation using these strategies.

This regime-dependent relation can be studied using Markov regime-switching models that are commonly used to explore regimes in financial time series, see Ang and Timmermann (2012) for an overview and Guidolin (2011) for an extensive review of applications of Markov-switching models in finance. We estimate a regime-switching Markov model driving a joint return process comprising the market portfolio excess return over the risk-free rate, the factor portfolios and a spread of the corresponding alternative beta strategy over the market portfolio. We then estimate a regime-dependent regression of alternative strategy excess returns⁴ on the market excess

returns and Fama-French factors - the size and the value.

The estimation procedure identifies four regimes, in line with Guidolin and Timmermann (2008), representing two low- and two high-volatility states⁵. The two low-volatility states differ by behavior of the factors, both in terms of realized returns and correlation to the market. The two high volatility regimes additionally differ by their persistence and the magnitude of the market return, corresponding to a moderate bear market and a crash state.

The main finding of this paper relates to the exposure of equal weight and minimum variance strategies to the size and value factors across regimes. We find that an equal weight strategy is generally positively aligned with the two factors, the size being the main driver of the strategy excess return in low-volatility states, and the value being more important in high-volatility environment. A very interesting pattern emerges for the minimum variance strategy. First of all, the strategy's exposure to the factors switches from negative to positive as regimes switch. But contrary to the equal weight strategy case, the discriminant factor for the factor exposure sign is not market volatility, but rather correlation between the factor portfolios and the market. In the states where the factors are negatively correlated to the market, the minimum variance strategy uses the opportunity to reduce variance by positive exposure to the factors. In the states with positive correlation between the market and the factors, minimum variance is exposed negatively to the factors. Thus, a minimum variance strategy rather uses the factors as

⁴Throughout the paper we denote by the term excess return a return above risk-free rate, while with the term spread we denote a return difference between two risky assets.

⁵In what follows, by low- and high-volatility states we mean states when the market portfolio volatility was low and high, respectively.

hedging instruments to partially offset market risk.

Thanks to advanced econometric tools, one can go beyond the standard linear regression and uncover interesting nonlinear properties of dynamic investment strategies. This article is, up to our knowledge, the first attempt to consider alternative beta strategies in regime-switching context. Our results complete the results of Chow et al. (2011), Scherer (2010) and Carvalho et al. (2012) that studied performance of alternative beta strategies using linear regression. We confirm that a significant relation exists between the alphas of alternative strategies and the size and value factors. But our study shows that one cannot ignore regime-dependent nature of this relationship. The time-varying factor exposures are in contradiction with the view that alternative beta strategies can be efficiently replicated by a static factor mix.

The paper is organized as follows: section 2 recalls empirical findings about equity market regimes, section 3 describes in more detail the two alternative strategies used in the paper, section 4 is devoted to the description of our model, the dataset and estimation of the model with regimes; sections 5–6 provide the empirical results for minimum variance and equal weight strategies; we conclude with section 7 in which we summarize our findings.

2 Regimes in equity markets

Empirical studies have showed that stock returns, volatilities and, when they exist, higher order moments, are strongly time dependent and exhibit features such as volatility clustering (Cont, 2001) and time-varying correlations (Longin and Solnik, 2001). An important stream of financial literature has

been devoted to the design and calibration of models able to capture these properties. The regime switching approach, introduced by Hamilton (1989) for modeling GNP growth series and extended later to financial markets, for example Turner et al. (1989) and Hamilton and Lin (1996), captures most of the stylized facts, while being analytically tractable and offering an intuitive interpretation of the regimes, relating the latter to economic cycles, bull and bear markets or important changes in market regulation.

In early applications, two-regime market models were used in the literature to model stock market returns (see Turner, Startz, and Nelson (1989), or Schaller and Van Norden (1997)). The main discriminating factor for the regimes appeared to be market volatility. Often average returns in different regimes are not significantly different, but the volatilities always are. Still, the returns in excess of the risk-free rate tend to be negative in high-volatility states and positive in low-volatility states. This allows to interpret the regimes as bull and bear states despite the poor significance of the means. Low-volatility bull regimes were found to be more persistent than high-volatility bear regimes, with only about 10% of the time spent in a bear market as shown in Maheu and McCurdy (2000).

However a larger regime set is needed when it comes to modeling a joint return process of several financial series. As Guidolin and Timmermann (2008) show, at least four regimes are needed to capture different patterns that emerge for asset pricing factors: market, size and value portfolios. In this case the regimes differ not only by volatilities of the market and factor portfolios, but also by their covariance. Three of the regimes correspond to bull markets, one of which is a transient regime with high volatility and very pronounced size

and value premiums. The two remaining bull markets are highly persistent, low-volatility states that differ in the way the factors are correlated to the market portfolio: in one state, the size portfolio provides a hedge for the performance of the market portfolio, and in the other state the value portfolio plays this role. Finally, Guidolin and Timmerman find one bear state that, apart from exhibiting high volatility of the market and factor returns, shows no significant value and size premiums.

Together with Markov models, the ARCH and GARCH models of Engle (1982) and Bollerslev (1986) and their extensions are valid candidates to capture time-dependence of asset price moments. Hamilton and Susmel (1994) and Hamilton and Lin (1996) introduced regime-switching models with ARCH terms. As is highlighted in Guidolin (2011), the regime-switching ARCH models are a better fit for daily and weekly data frequency, whereas for monthly data ARCH effects are less clear. However, to keep the setup simpler and to avoid estimating a high number of parameters we do not use an autoregressive component in this study.

3 Alternative Beta Strategies

The alternative beta name is used for portfolios that are meant to be passive investment benchmarks that add extra value on top of the traditional beta - market portfolio - without using active management. As such, they are broad, passively managed and representative of a certain investment universe, i.e. are not restricted to some specific investment theme or risk factor (e.g., a bet on an industrial sector or a value portfolio). A common theme in the alternative beta space is to correct inefficiencies of market cap-weighted portfolios, such

as over-concentration on mega-cap stocks and trend-following. This is achieved by employing systematic rules in portfolio construction, that aim at adding to the broad market beta an extra (risk-adjusted) benefit. It is common to distinguish several subgroups within the alternative beta family: risk-based approaches incorporating risk management objectives in portfolio construction (such as minimum variance and risk parity), fundamental-based strategies that attempt to build improved return forecasts from micro- and macroeconomic data (such as Fundamental index⁶ and weighting by GDP), and diversification-based approaches that ignore completely return as well as risk forecasts and concentrate on smoothing the distribution of portfolio weights (such as equal weight approach). We do not attempt here to give an exhaustive description of all alternative beta strategies and address the reader to Monnier and Rulik (2012) for more information.

We study here two representatives of the alternative beta family: equal weight and minimum variance strategies. An equal-weight portfolio spreads investment bets evenly across the investment universe. This is an optimal strategy if one searches for maximal diversification in absence of reliable information on stocks' future risk and returns. Naturally, an equal-weight portfolio corrects the mega-cap concentration bias of market cap-weighted portfolios and avoids trend-following behavior, as the portfolio is rebalanced periodically to restore equal weights and the past outperformance of a stock relative to the basket does not lead to a superior weight for it. Consequently, an equal-weight portfolio will not follow a market bubble.⁷ Risk profile of

⁶The concept is created by Research Affiliates.

⁷With 62 Internet companies in the S&P index in late 1999, the total weight of the IT sector in the S&P 500 Equal Weight was of the order of $62/500 = 12.4\%$,

Strategy	S&P 500	Equal Weight	Minimum Variance
Total return	241.2%	511.69%	234.33%
Annualized return	6.41%	9.6%	6.3%
Volatility	18.72%	19.51%	12.99%
Maximal drawdown	-56.78%	-59.77%	-38.14%
Sharpe ratio	0.3425	0.4924	0.4854
Beta		1.005	0.62
Spread benchmark		3.19%	-0.11%
Tracking error		5.15%	9.21%

Table 1: Return characteristics of alternative beta strategies on S&P 500 universe. Oct 1991-Jul 2011

an equal-weight portfolio resembles that of the cap-weighted portfolio. Equal weighting brings neither significant increase nor decrease in portfolio long-term volatility with respect to the market-cap counterpart, and an equal-weight portfolio generally remains very correlated to the cap-weighted portfolio (see e.g. Carvalho et al. (2012)).

A minimum variance portfolio is an optimal portfolio constructed by an optimization procedure aimed at minimizing portfolio variance. Minimum variance portfolio construction does not use expected returns as inputs, and relies only on an estimated covariance matrix. This approach allows significant reduction of portfolio risk⁸. More details on the strategy can be found in Monnier and Rulik (2011). Contrary to equal weighting, where portfolio construction is straightforward and unambiguous, a minimum variance approach allows to add extra value via careful optimization design. Because of subtleties in covariance estimation and the choice of constraints to mitigate covariance estimation error, minimum variance methodologies differ

significantly from one provider to another. Minimum variance portfolios can be very basic, including only full-investment and perhaps long-only constraints with a simple historical estimate used for the covariance matrix; one encounters such "plain-vanilla" minimum variance portfolios in empirical research papers. Or, a minimum variance approach can be enriched with a multitude of constraints and complex risk models, as is the case of existing minimum variance indices. Both extremes have their drawbacks. Over simplistic implementation leads to highly concentrated portfolios and has no control over portfolio liquidity, which is unacceptable for an investment benchmark. A heavy set of constraints, especially binding the portfolio weights to resemble the market portfolio weights, conflicts with the very objective of portfolio variance minimization. In this paper we use a minimum variance methodology that in our opinion strikes a reasonable balance between the pursuit of minimal possible variance and addressing the estimation error issue and liquidity concerns.⁹ Optimization is based on an empirical covariance matrix with

that was much lower than the 29.2% weight of this sector in the S&P 500 portfolio.

⁸Volatility reduction of 30% is the number often cited by different sources, see e.g. Clarke et al. (2006).

⁹A similar methodology is used in several existing minimum variance indices: iSTOXX Europe Minimum Variance, FTSE 100 Minimum Variance and a family of Ossiam Minimum Variance custom indices.

the following constraints: full investment, no short sales, maximum sector exposure of 20% and a quadratic diversification constraint restricting the portfolio 2-norm to be equal to $1/50$. The last constraint allows to target a desired level of diversification and improves implicitly the quality of covariance estimation, as it is equivalent to performing covariance matrix shrinkage (DeMiguel et al. (2009)). The stocks are screened by their liquidity and only the most liquid half of the universe is considered in portfolio optimization. The portfolio is rebalanced on a monthly basis.

In this study we analyze alternative beta strategies on large-cap US stocks, for the period from October 1990 to July 2011. We take an S&P 500 Equal Weight index¹⁰ which invests the same amount of $1/500$ in each stock of the S&P 500 index and resets its allocation quarterly. We also build minimum variance portfolio on the S&P 500 universe according to the methodology outlined above¹¹.

¹⁰Source: Bloomberg

¹¹The S&P 500 composition data is a courtesy of S&P. The S&P 500 Index ("Index") is a product of S&P Dow Jones Indices LLC and/or its affiliates and has been licensed for use by Ossiam. Copyright ©201[x] by S&P Dow Jones Indices LLC, a subsidiary of the McGraw-Hill Companies, Inc., and/or its affiliates. All rights reserved. Redistribution, reproduction and/or photocopying in whole or in part are prohibited without written permission of S&P Dow Jones Indices LLC. For more information on any of S&P Dow Jones Indices LLC's indices please visit www.spdji.com. S&P® is a registered trademark of Standard & Poor's Financial Services LLC and Dow Jones ® is a registered trademark of Dow Jones Trademark Holdings LLC. Neither S&P Dow Jones Indices LLC, Dow Jones Trademark Holdings LLC, their affiliates nor their third party licensors make any representation or warranty, express or implied, as to the ability of any index to accurately represent the asset class or market sector that it purports to represent and neither S&P Dow Jones Indices LLC, Dow Jones Trademark Holdings LLC, their affiliates nor their third party licensors shall have any liability for any errors, omissions, or interruptions of any index or the data included therein.

In Table 1 we report the performance summary of the strategies along with the market capitalization weighted S&P 500 index. Both alternative strategies offer risk-adjusted return superior to the S&P 500 index, but they achieve this superior performance in different ways. The equal weight strategy does it via its stock-picking ability and has roughly the same level of risk as the market capitalization benchmark, while the minimum variance strategy significantly reduces portfolio volatility and draw-down without reduction in portfolio performance.

4 Methodology

4.1 Data

We form a dataset comprising: a market portfolio (the S&P 500 index), an equal weight portfolio, a minimum variance portfolio and Fama-French size and value portfolios extracted from the Kenneth French website. The Fama-French size and value factors correspond to returns on long-short portfolios that are long the stocks with low attributes (such as market capitalization for the size portfolio and book-to-market ratio for the value portfolio) and short the stocks with high attributes.

The returns in the dataset are taken on a weekly scale, that gives 1028 weekly observations from October 1991 to July 2011 for each series. Note that the bulk of regime-switching literature uses monthly returns for model estimation. Weekly frequency is used less often since it is believed to obscure the structure in the conditional mean and lead to less persistent regimes. However, some recent papers (see e.g. Maheu et al. (2011)) showed that the stock market models with switching on weekly data can be more persistent in the context of a 4-state model that distinguished

between long-term trends and shorter-term corrections. Weekly data frequency is also used extensively in the models with extended sets of parameters that should be estimated, such as MS GARCH models. Weekly data allows us to extract more significant results for relatively short time periods. Only a small number of macro-economic and index series are available for suitably long periods of time (e.g. inflation, US equity indices, gold,..), as datasets of more than 50 years routinely used in regime-switching literature. For the alternative beta strategies used in this study, we could provide reliable backtests for a much shorter period: about 20 years.

In what follows, we form two distinct datasets, each including a spread of one alternative beta strategy (either minimum variance or equal weight) over the market portfolio, along with the market excess return over a risk-free rate and two Fama-French factors (size and value). Further, we denote by SPEW and SPMV the spreads of respective alternative strategies over the market, by MKT the market portfolio excess return (S&P 500 index minus T-bill rate) and by HML (High Minus Low) and SMB (Small Minus Big) value and size factors, respectively. These datasets are then used to calibrate regime-switching multivariate return models. Consequently, regimes that are induced from the data are endogenous with respect to the alternative beta strategies under study (the strategy takes part in the calibration process and thus influences the regime pattern). This, in our opinion, helps to uncover more precisely the properties of the strategies, so their behavior is not averaged across exogenously imposed regimes. In this way we also get separate sets of regimes for equal weight and for minimum variance strategies, even though the two sets have very similar characteristics.

4.2 Regime-switching model of portfolio returns

We choose to model the joint distribution of returns with a multivariate regime switching model where regimes are governed by a common one-dimensional discrete state variable. Let r_t be the $n \times 1$ vector of returns and S_t the state variable, then our model is :

$$r_t = \mu_{S_t} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma_{S_t})$$

where μ_S is a state-dependent mean of returns. Likewise, Σ_S is the returns' covariance matrix in the state S . The vector of innovations ε_t is thus a mixture of multivariate normal distributions.

The state variable S_t follows a discrete first order Markov chain with values in $[1, \dots, k]$ where k is the number of regimes. Its dynamic is determined by its $k \times k$ transition probability matrix P , whose elements are defined by

$$p_{j,i} = \mathbb{P}(S_t = i | S_{t-1} = j), \quad i, j = 1, \dots, k$$

S_t is not directly observable and we estimate it by a smoothing procedure.

4.3 Estimation of regimes

Estimation is performed using the Expectation-Maximization (EM) algorithm as described in Elliott et al. (2008). At the expectation step, we determine a smoothed estimate of S_t - which is unobservable - given our current estimates of the model parameters. In the optimization step, we update our parameter estimate via maximum likelihood optimization while treating our smoothed estimate of the state variable as a direct observation of S_t .

In order to select the number of regimes to use in our analysis, we test different specifications for k ranging from 1 to 4. We then choose the specification which is the most relevant according to the Bayesian Information Criteria (BIC). For our datasets of 4 series each, every additional regime implies estimation of 14 extra parameters (4 for the mean, 4 for the variance and 6 for the correlations) plus an additional transition probability. The BIC penalizes over-parameterized models allowing to select a specification that is relevant and parsimonious at the same time.

For both datasets this test elects a 4-regime model. In Sections 5 and 6 we will describe in detail the regimes for each alternative beta strategy.

4.4 Regression analysis

Once the regimes are defined, we perform a linear regression of the alternative beta strategy excess returns over the MKT and the Fama-French factors. We use the corresponding alternative strategy excess return as a dependent variable and the factors MKT, SMB and HML as explanatory variables:

$$\begin{aligned} r_t^{strategy} = & \alpha(S_t) + \beta^{MKT}(S_t)r_t^{MKT} \\ & + \beta^{SMB}(S_t)r_t^{SMB} \\ & + \beta^{HML}(S_t)r_t^{HML} + \\ & + \omega(S_t) \end{aligned}$$

Regression estimates can be computed within the EM setup as part of the maximization step. Coefficients within each regime are moments conditioned on the state variable S_t . Within the maximization step, we use the distribution probability of smoothed estimator of S_t , \hat{S}_t , obtained in our calibration process. By using the observed estimate instead of the unobservable state variable, a classic linear

regression is then performed.

In a model setup without regimes, the parameter vector $(\alpha, \beta^{MKT}, \beta^{SMB}, \beta^{HML})$ can be seen as a conditional expectation in which every observation $(r_t^{MKT}, r_t^{SMB}, r_t^{HML})$ in the dataset has the same weight (1/number of observations). The same intuition holds true when using regimes, but in this case all the observations are weighted by probability of being in a given regime of the Markov chain. For example, to estimate the parameter vector on the Regime $S = 1$, we use the whole dataset but each observation $(r_t^{MKT}, r_t^{SMB}, r_t^{HML})$ used in the estimation procedure receives a weight proportional to the probability of being in the State 1.

5 Empirical results for minimum variance

In this section we report results of regime estimation and in-regime regression for the minimum variance strategy, following the methodology described in Section 4. We recall that regimes are calibrated on a dataset including the alternative beta strategy spread over the market portfolio (SPMV), the market portfolio (MKT) and the size and value factors (SMB and HML).

As we already mentioned, a four-regime model was retained as the most statistically suitable one according to the BIC criteria. Table 2 presents the return means, volatilities and correlations inside each regime, as well as transition probabilities for the regimes. Figure 1 depicts historical probabilities of being in each regime. The means and volatilities reported are annualized.

We observe two bull and two bear regimes, with the bull regimes (1 and 3) having lower

Annualized expected return		MKT	SPMV spread	SMB	HML
	Regime 1	0.1266	-0.0243	-0.0339	0.0557
	Regime 2	-0.0174	-0.0178	0.0764	0.0851
	Regime 3	0.0985	-0.0008	0.0480	0.0114
	Regime 4	-0.2256	0.0529	0.0151	0.0180
Correlations/Volatilities					
Regime 1 bull pre 2000	MKT	0.1107			
	SPMV spread	-0.5913	0.0482		
	SMB	-0.4269	0.2528	0.0715	
	HML	-0.3889	0.4438	-0.2666	0.0710
Regime 2 IT bubble	MKT	0.1955			
	SPMV spread	-0.7509	0.1304		
	SMB	-0.0589	-0.1439	0.1358	
	HML	-0.6258	0.7839	-0.3874	0.1426
Regime 3 bull post 2000	MKT	0.1192			
	SPMV spread	-0.7544	0.0515		
	SMB	0.3199	-0.4065	0.0750	
	HML	0.0378	-0.0443	-0.0534	0.0508
Regime 4 crisis	MKT	0.3409			
	SPMV spread	-0.7796	0.1396		
	SMB	0.0778	-0.1883	0.1113	
	HML	0.6461	-0.8089	0.0884	0.1809
Transition Probabilities		Regime 1	Regime 2	Regime 3	Regime 4
	Regime 1	0.9940	0.0064	0.0000	0.0000
	Regime 2	0.0060	0.9704	0.0081	0.0316
	Regime 3	0.0000	0.0118	0.9741	0.0601
	Regime 4	0.0000	0.0114	0.0177	0.9083

Table 2: Four State Model for SPMV spread and the Fama-French factors. The central table represents a mixed volatility/correlation matrix, with the diagonal elements being volatilities and off-diagonal elements being pairwise correlations. Expected returns and volatilities are annualized.

market volatility than the bear regimes (2 and 4). Similar to the results of Guidolin and Timmermann (2008), we find that the size and value factors are positively correlated to the market portfolio in one of the two low-volatility regimes and negatively correlated in the second low-volatility regime. Similarly, the two high-volatility regimes also differ (among other things) by correlation between the factors and the market portfolio. Magnitude and sign of value and size premiums also vary with

regimes: the value effect is more persistent and especially pronounced in regimes with negative correlation between this factor and the market; the size premium is positive and significant in two out of four regimes (one bull and one bear) and does not seem to be related to a specific correlation pattern.

Regime-dependent statistics of the SPMV spread indicate that minimum variance strategy average returns are smaller than that of

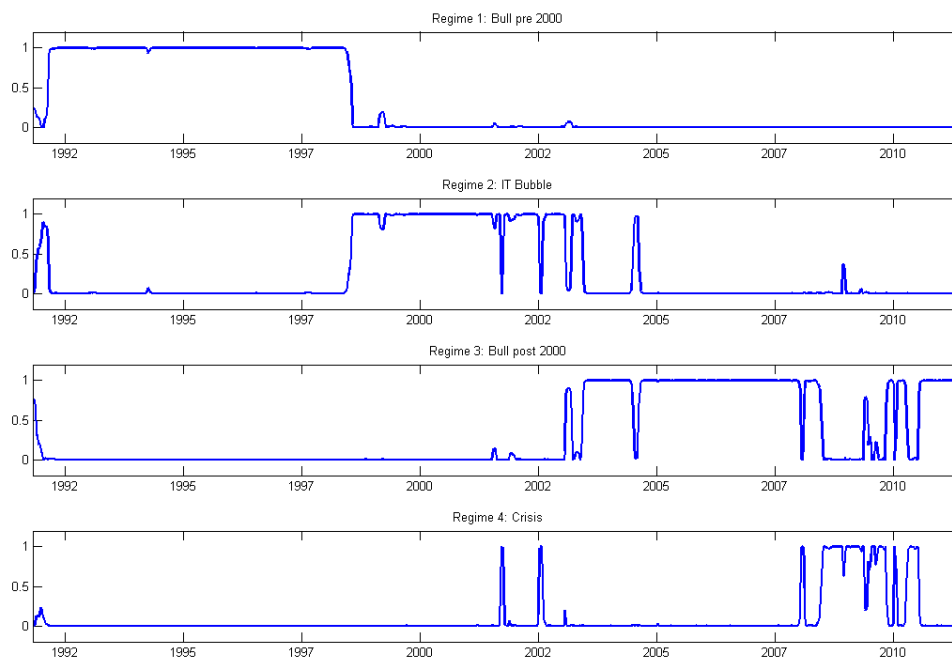


Figure 1: Probabilities of different regimes for the minimum variance dataset

the market portfolio in three out of four states (two bull states and a moderate bear state). The SPMV spread, that represents outperformance of the minimum variance strategy over the market portfolio, is always negatively correlated to the market and changes its correlation to the size and value factors depending on regimes. It tends to have significant positive correlation to a factor when the factor is itself negatively correlated to the market and vice versa. In the practitioners' language, SPMV spread volatility corresponds to a tracking error between the minimum variance portfolio and the market index¹². This measure is closely monitored, as many managers have explicit restrictions on deviation from the

market benchmark. As shown in the Correlation/Volatilities panel of the Table 2, higher tracking error is detected in the high-volatility states 2 and 4 and was above 13%. In the low-volatility bull markets 1 and 3 the tracking error of the minimum variance strategy was modest, up to 5% annualized.

Below we give a detailed description of each regime.

Regime 1 is a highly persistent low volatility bull state whose average duration is 166.7 weeks. In fact this regime is present almost continuously from October 1991 to July 1998 (its end coincides with the Russian debt crisis). It thus captures most of the bull market prior to the Internet Bubble¹³. This state exhibits a negative correlation between the market portfolio

¹²The tracking error is defined as standard deviation of the return differences between the portfolio and the market

¹³This regime has an impressive persistence that ex-

and the size and value portfolios. The value factor had significant positive premium (+5.57% of expected return), while the small capitalizations underperformed large capitalizations (the long-short SMB portfolio has a return of -3.39%).

Regime 2 is a persistent bear state whose average duration is 33.7 weeks and it spans the Internet bubble run-up together with the 2000-2002 crisis. On average, the market is slightly down in this regime (-1.74%) but is highly volatile (19.55%). The main peculiarity of this regime is a strong negative correlation between the market and the value portfolio. Both the size and the value exhibit positive returns (7.64% and 8.51%, respectively).

Regime 3 is an additional persistent low volatility bull state. Its average duration is 38.6 weeks. This regime captures most of the 2003-2007 bull period and the recent recovery since 2011. There is marked difference in the behavior of the HML and SMB factors with respect to the bull Regime 1. The factor portfolios show zero or positive correlation to the market, with SMB delivering positive risk premium this time, as opposed to its behavior in the 1990-2000 bull market.

Regime 4 is a highly volatile state whose average duration is 10.9 weeks. It is a resolutely bear market (-22.42% of annualized return). It appears briefly during the post-bubble crisis of 2001-2002 and covers the 2008 crisis. It is characterized by positive returns for the SMB, HML portfolios and the SPMV spread at the same time.

ceeds the results obtained in previous literature that uses four-regime specification. This might need further investigation as the regime appears only once during the period and could indicate a presence of a structural break.

The value factor is strongly correlated to the market in this state.

Table 3 presents results of factor regression of minimum variance excess returns on the Fama-French Factors within the regimes, as well as for the whole period. The intercepts α are not significant within the regimes¹⁴, but the global regression gives small positive alpha of 0.52% annually.

Factor betas are significant across all the regimes, as well as in the global regression. The only beta with significance level below 0.05 corresponds to value exposure in the regime 3. In accordance with Chow et al. (2011) we find that over the whole period the minimum variance is positively exposed to the value HML factor, whereas we find significant negative exposure to the size factor while Chow et al. (2011) document no significant contribution from the size factor. This can be explained by the fact that minimum variance portfolios tested in their study were constructed using a different version of minimum variance methodology. In particular, the minimum variance construction that we have used here employs a liquidity filter to eliminate the less liquid half of the investment universe before running the minimum variance optimization. This step is not present in minimum variance backtests used in other papers and naturally creates a portfolio less exposed to small cap stocks.

Market beta for the minimum variance strategy is stable across the regimes, ranging from 0.7 to 0.835. By contrast, factor exposure varies across regimes, being positive in the regimes 1 and 2 and negative in the regimes 3 and 4. The following pattern emerges when comparing the tables 3 and 2: in regimes

¹⁴significance level: *** is 0.05; ** is 0.1; * is 0.15; otherwise not significant at 0.15 level.

Data range	α	MKT	SMB	HML
All	0.0001**	0.6362***	-0.1027***	0.1316***
Regime 1 (bull pre 2000)	-0.0003	0.8352***	0.1233***	0.2361***
Regime 2 (IT bubble)	-0.0013	0.7219***	0.0419***	0.5003***
Regime 3 (bull post 2000)	0.0006	0.7012***	-0.1256***	-0.0312**
Regime 4 (crisis)	0.0002	0.8277***	-0.1438***	-0.4025***

Table 3: Regression Result for SPMV excess return

where the size and the value factor returns are negatively correlated to the market portfolio, the minimum variance strategy goes long these factors and this allows it to further reduce portfolio volatility. Vice versa, in regimes when the factors move together with the market, the minimum variance strategy becomes exposed negatively to these factors, implementing thus, once again, a hedge against market risk. This is a remarkable and consistent finding that intuitively makes sense since the strategy has a clear objective of volatility reduction (while maintaining full investment in equities) and is allowed to reposition the portfolio frequently on the basis of recent data on stocks' covariance. As such, it captures temporary hedging opportunities offered by the factors.

6 Empirical results for equal weight

As before, we first determine market regimes on a dataset including the spread of an equal weight strategy over the market (we denote it by SPEW), as well as the market and the factor portfolios. The regimes appear to be similar to those of the minimum variance dataset, though their timing somewhat differs (the bulk of the regimes are overlapping, though not perfectly). We report regime statistics in the table 4 and show evolution of regime probabilities on the Figure 2.

Regimes on the equal weight dataset are less persistent than those on the minimum variance dataset. We detected two bull and two bear regimes, with market volatility being low in the bull phases and high in the bear phases. The discriminant factors among different bull and bear regimes are, once again, correlations among factor portfolios and the market, as well as the factor premiums. One sees that the equal weight spread SPEW volatility, that is equivalent to the tracking error of the strategy, is smaller during the bull markets (3-3.5%), while rising sharply in the bear markets (up to 14%), as we also found for the minimum variance spread. The sign of the equal weight spread is instead positive in all regimes but the bull regime 1, suggesting that an equal weight strategy adds value by outperforming the market rather than delivering risk reduction as a minimum variance strategy does.

Regime 1 is a persistent low volatility bull state whose average duration is 26.2 weeks. It is a dominant regime for the period 1991-1998. This regime captures most of the bull market prior to the Internet Bubble. Nonetheless, it also appears for shorter periods in 2001-2004 and in 2008-2009. It exhibits a significant anti-correlation between the SML, HMB and MKT portfolios.

Regime 2 is a non-persistent state whose av-

Expected return		MKT	SPEW spread	SMB	HML
	Regime 1	0.1183	-0.0256	-0.0299	0.0254
	Regime 2	-0.0804	0.1525	0.1895	0.1667
	Regime 3	0.0867	0.0400	0.0352	0.0322
	Regime 4	-0.1658	0.0020	-0.1474	-0.1058
Correlations/Volatilities					
Regime 1 bull pre 2000	MKT	0.1277			
	SPEW	-0.2134	0.0352		
	SMB	-0.4912	0.5486	0.0721	
	HML	-0.3870	0.2347	-0.1786	0.0709
Regime 2 IT bubble	MKT	0.1708			
	SPEW	0.0111	0.0721		
	SMB	0.0565	0.1481	0.1001	
	HML	-0.5087	0.5242	-0.3192	0.1306
Regime 3 bull post 2000	MKT	0.1215			
	SPEW	0.4672	0.0299		
	SMB	0.4312	0.7412	0.0779	
	HML	0.2291	0.2282	0.0243	0.0616
Regime 4 crisis	MKT	0.4099			
	SPEW	0.2020	0.1442		
	SMB	-0.0265	-0.0128	0.2088	
	HML	0.2077	0.7477	-0.3437	0.2350
Transition Probabilities		Regime 1	Regime 2	Regime 3	Regime 4
	Regime 1	0.9619	0.0590	0.0061	0.0090
	Regime 2	0.0298	0.8538	0.0021	0.2469
	Regime 3	0.0039	0.0038	0.9919	0.0198
	Regime 4	0.0044	0.0834	0.0000	0.7243

Table 4: Four State Model for SPEW spread and the Fama-French factors. The central table represents a mixed volatility/correlation matrix, with the diagonal elements being volatilities and off-diagonal elements being pairwise correlations. Expected returns and volatilities are annualized.

erage duration is 6.8 weeks. It captures the IT Bubble period with both the run up and the draw down. This regime is characterized by strong negative correlation of the value factor to the market (-50.87%), as growth high-tech stocks dominated the market performance. At the same time there was positive correlation between SPEW spread and the value factor, and almost no correlation of the SPEW spread to the market, indicating

that the equal weight portfolio was also less exposed to high-tech stocks.

Regime 3 is a highly persistent low-volatility bull state with average duration of 123.5 weeks. The market and size and value factors are all positively correlated and are trending upward. This regime thus exhibits reduced diversification potential, and the equal weight strategy is fairly correlated to the market and to the factors in

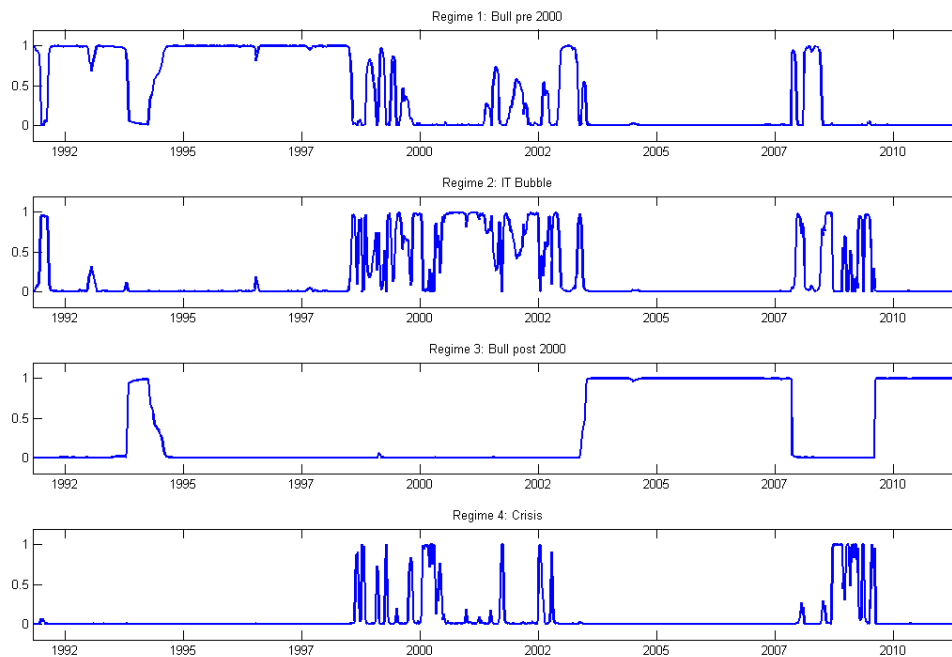


Figure 2: Probabilities of different regimes for the Equal Weight dataset

this regime, with the biggest correlation to the size factor.

Regime 4 is a highly volatile, transient state whose average duration is 3.6 weeks. It captures strong up and down movements in market returns during 1998-2002 and in 2008-2009. The MKT return is strongly negative in this state and the SMB and HML factors both deliver negative returns. The SPEW spread is strongly correlated to the value factor here.

The results of regression of equal weight excess returns on the factors within the regimes is given in the Table 6. Market as well as factor betas are all significant across regimes, as well as over the global dataset. The equal weight strategy is consistently positively exposed to the size and value factors. The magnitude of

this exposure varies across regimes, with the size factor being more pronounced in the bull regimes 1 and 3, with the value factor being dominant in the bear regimes 2 and 4. In the regime 2, the value factor is negatively correlated to the market, thus the equal weight strategy has lower volatility in this state, as market risk is partially offset.

7 Conclusion

We studied the regime-dependent behavior of two alternative beta strategies: minimum variance and equal weight, and, in particular, the relationship between their performance and the size and value factors across different regimes. Both strategies deliver risk-adjusted benefit with respect to the market portfolio, as was already documented in previous litera-

Data range	α	MKT	SMB	HML
all	0.0002***	1.0575***	0.2393***	0.3603***
Regime 1 (bull pre 2000)	-0.0007***	1.1245***	0.4290***	0.2850***
Regime 2 (IT bubble)	0.0011***	1.1578***	0.2390***	0.4304***
Regime 3 (bull post 2000)	0.0005***	1.0318***	0.2671***	0.0875***
Regime 4 (crisis)	0.0027***	1.0011***	0.2211***	0.5582***

Table 5: Regression for the SPEW spread

ture. We also confirm in our study that over long time periods there are significant contributions of the pricing anomalies like size and value in the strategies' performance. However on shorter horizons there are important changes in the factor exposures: a Markov switching model detects four statistically distinct regimes that differ by market volatility, magnitude of size and value premiums and correlation among the strategies, the factors and the market.

It was already documented in previous literature that pricing anomaly factors themselves have a complicated covariance pattern with respect to the market portfolio, that varies with regimes. In particular, Guidolin and Timmermann (2008) found that there exist several distinct regimes, differing by the way the size and value factors are correlated to the market: in some regimes the factors are positively correlated to the market and in other regimes this relation is inverse. We confirm this pattern in our study.

In addition, we find that an equal weight strategy has moderate variability of factor exposure across regimes. It is always positively exposed to the size and value factors and it is only the relative importance of these factors that varies across regimes. Notably, the size factor is the most important explanation factor in bull regimes, while the value factor is dominant in bear regimes.

The most interesting pattern emerges for a

minimum variance strategy. Factor exposures vary greatly for this strategy, moving from negative to positive in different regimes. The exposure of a minimum variance strategy to the size and value factors is determined by a hedging potential the factors can represent in some regimes. Indeed, when the size and the value factors are negatively correlated to the market, a minimum variance strategy is exposed positively to these, and vice versa if the size and the value factors move together with the market. This is exactly what the minimum variance engine is supposed to do: find factors that are negatively correlated to the broad market and use them to reduce portfolio variance.

Our results show that alternative beta strategies can have dynamic exposure to pricing factors and should not be reduced to a simple linear factor mix. These findings should be considered when including alternative beta strategies in a global portfolio, or creating an allocation combining "smart" beta strategies and style portfolios.

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References

- Ang, A. and A. Timmermann (2012). Regime changes and financial markets. *Annual Review of Financial Economics* 4, 313–337.
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* 31, 307–327.
- Carvalho, R., X. Lu, and P. Moulin (2012). Demystifying equity risk-based strategies: A simple alpha versus beta description. *Journal of Portfolio Management* 38(3), 56–70.
- Chow, T., J. Hsu, V. Kalesnik, and B. Little (2011). A survey of alternative equity index strategies. *Financial Analyst Journal* 67(5), 37–57.
- Clarke, R., H. de Silva, and S. Thorley (2006). Minimum-variance portfolios in the us equity market. *Journal of Portfolio Management* 33(1), 10–24.
- Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance* 1, 223–236.
- DeMiguel, V., L. Garlappi, N. J., and U. R. (2009). A generalized approach to portfolio optimization: improving performance by constraining portfolio norms. *Management Science* 55, 798–812.
- DeMiguel, V., L. Garlappi, and R. Uppal (2009). Optimal versus naive diversification: how inefficient is the 1/n, portfolio strategy? *The review of Financial Studies* 22(5), 1915–1953.
- Elliott, R., L. Aggoun, and J. Moore (2008). *Hidden Markov models: estimation and control* (Second ed.), Volume 29. Springer.
- Engle, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of united kingdom inflation. *Econometrica* 50(4), 987–1008.
- Fama, E. and K. French (2004). The capital asset pricing model: theory and evidence. *Journal of Economic Perspectives* 18(3), 26–46.
- Guidolin, M. (2011). Markov switching models in empirical finance. *Working paper, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University* (415).
- Guidolin, M. and A. Timmermann (2008). Size and value anomalies under regime shifts. *Journal of Financial Econometrics* 6(1), 1–48.
- Hamilton, J. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*, 357–384.
- Hamilton, J. and G. Lin (1996). Stock market volatility and the business cycle. *Journal of Applied Econometrics* 11(5), 573–593.
- Hamilton, J. and R. Susmel (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics* 64, 307–333.
- Longin, F. and B. Solnik (2001). Extreme correlation of international equity markets. *Journal of Finance* 56, 649–676.
- Maheu, J. and T. McCurdy (2000). Identifying bull and bear markets in stock returns. *Journal of Business & Economic Statistics*, 100–112.
- Maheu, J., T. McCurdy, and Y. Song (2011). Components of bull and bear markets: bull corrections and bear rallies. *University of Toronto, Working Paper*.

- Markowitz, H. (2005). Market efficiency: a theoretical distinction and so what? *Financial Analysts Journal* 61(5), 17–30.
- Monnier, B. and K. Rulik (2011). Mechanics of minimum variance investment approach. *Working Paper*.
- Monnier, B. and K. Rulik (2012). Efficient portfolio: Market beta and beyond. *ETFs and Indexing* 2012(1), 68–80.
- Plyakha, Y., R. Uppal, and G. Vilkov (2012). Why does an equal-weighted portfolio outperform value- and price-weighted portfolios? *Working paper*.
- Schaller, H. and S. Van Norden (1997). Regime switching in stock market returns. *Applied Financial Economics* 7(2), 177–191.
- Scherer, B. (2010). A new look at minimum variance investing. *SSRN 1681306*.
- Turner, C., R. Startz, and C. Nelson (1989). A markov model of heteroskedasticity, risk, and learning in the stock market. *Journal of Financial Economics* 25(1), 3–22.

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