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***Carmine De Franco, PhD***  
Quantitative analyst  
carmine.de-franco@ossiam.com

***Bruno Monnier, CFA***  
Quantitative analyst  
bruno.monnier@ossiam.com

***Ksenya Rulik, PhD, CFA***  
Head of Quantitative Research  
ksenya.rulik@ossiam.com

## Abstract

We assess the exposure of stock portfolios sorted by total volatility to interest rate risk and determine whether this non-equity risk can explain differences in risk and risk-adjusted returns between low and high volatility portfolios over a 20-year period for the US equities. We find that the addition of interest rate risk factor to the 4-Factor model reveals small but positive duration for low-volatility portfolios. However this new factor fails to improve the explanatory power of the model for both low and high volatility portfolios, and has no significant impact on the portfolios' risk-adjusted return. We find that interest rate factor loadings are fairly robust across different specifications of the multi-factor model for low-volatility portfolios but are unstable for high-volatility portfolios. For all volatility portfolios under study, the significance of the results is highly dependent on the choice of the time period.

**Key words:** Low volatility investing; Equity duration; CAPM; Fama-French equity model;

## 1 Introduction

Low volatility investing has recently gained particular attention. On one hand, the uncertain market environment poses questions on whether the equity investments can be accessed with less risk. Low volatility portfolios provide just this: minimum variance portfolios, for example, were shown to reduce volatility by 30% with respect to the market cap benchmark<sup>1</sup>. On the other hand, empirical evidence was gathered of long-term out-performance of low volatility stocks with respect to high volatility stocks (Haugen and Heins, 1975; Haugen and Baker, 1991; Ang et. al, 2006; Haugen and Baker, 2010). This undermined the widespread belief that higher risk is necessarily rewarded with higher expected returns, as follows from the Capital Asset Pricing Model.

Recent literature has mainly concentrated on attributing the long-term outperformance of low volatility portfolios to their exposure to well-known equity risk factors such as small cap, value and momentum. Chow et al. (2011) show that a whole family of equity strategies, that go by the name of alternative beta, have significant exposure to these rewarded factors. Yet, in this framework the puzzle of abnormal performance of low volatility stocks remains unsolved: even when accounting for market and Fama-French-Carhart factors, almost one third of the return variability of low volatility portfolios remains unexplained, as well as their superior risk-adjusted return. As a matter of fact, different ver-

sions of "volatility" risk factor that carries a positive premium have been recently proposed to explain the abnormal excess returns of low-volatility portfolios: Baker et. al (2011); Baker and Haugen (2012); Carvalho et. al (2012); Sullivan et al. (2012). Yet, from the point of view of asset pricing theory, this risk factor is very unusual as it rewards the reduction of overall equity risk, in direct contrast with traditional risk factors such as value, momentum and size. One alternative approach is to consider that low volatility portfolios, that are rebalanced frequently and react to the changes in the market, have non-linear relation to the traditional equity factors. Using regime-switching regressions, De Franco et al. (2013) showed that factor exposure of minimum variance portfolio is not static and changes across market regimes.

Alternatively, several authors proposed to resolve the paradox of volatility anomaly by assuming that low volatility stocks expose investors to non-equity risks. For example, Karyda et al. (2014) propose to see low volatility stocks as investments that are in between stocks and bonds, thus exposing investors to interest rate risk. The intuition behind comes from the classic Merton's capital structure theory (Merton, 1974). Since equity can be seen as a call-option over the firm's asset value, it should have negative duration (as firm's debt has positive duration, Choi et al. (2014)). As different companies have different capital structures, so their equities may have different sensitivity to interest rate movements. The question is then to assess if volatility can discriminate among companies with different capital structure and, as a consequence, low volatility focus might be responsible for selecting companies whose equity is more

<sup>1</sup>For example, the MSCI Minimum Volatility World index backtested back to 1995 had around 30% reduction in volatility with respect to the market-capitalization weighted index over the period 1995-2007, as documented by Nielsen and Aylursubramanian (2008)

sensitive to interest rate movements.

Another argument in the same direction is that low volatility portfolios have risk profile similar to a mix of, for example, 70% equity and 30% fixed income (Poullaouec (2008)) and thus can be considered as a substitute for an equity-bond allocation. Higher interest rate exposure might also stem from higher exposure of low volatility portfolios to such sectors as Utilities and Telecoms that historically showed higher sensitivity to interest rate shocks (Guissi, 2014).

If the interest rate risk factor is responsible for the abnormal performance of low volatility stocks, the three following Statements must hold true:

**Statement 1** The inclusion of the interest rate factor should significantly improve the explanation power of the standard Fama-French-Carhart factor model at least for low volatility portfolios.

**Statement 2** The inclusion of the interest rate factor should significantly reduce the alpha generated by low volatility portfolios.

**Statement 3** The empirical duration of low volatility portfolios should be positive and significant.

The aim of this paper is to estimate the exposure of portfolios with different level of volatility to interest rate risk and to investigate what portfolio characteristics might be responsible for this interest rate sensitivity. We take an empirical approach and look at the capability of the interest rate risk factor to explain a part of the performance and risk of these portfolios, not captured by standard equity models (as the CAPM or

4-Factor Fama-French-Carhart model). We use several alternative factor model specifications to measure equity duration for portfolios with different level of volatility to see if a stable pattern in relative duration of high volatility and low volatility equity investments can be confirmed.

A caveat in building empirical estimates of equity sensitivity to interest rates is that the latter strongly depend on the specification of the model used to measure such sensitivity (see for ex. Cornell (1999)). We find that the addition of an interest rate risk factor to the 4-Factor model gives very little improvement of explanatory power, both for high volatility portfolios (already fairly well explained by the 4-Factor model with  $R^2$  over 80%) and for low volatility portfolios (where the model fails to capture a significant portion of the portfolios' total variation). Moreover, the addition of the interest rate risk factor does not shrink significantly the risk-adjusted returns generated by low volatility portfolios and cannot explain performance difference between low volatility and high volatility stocks. We thus reject the Statements 1 and 2.

Empirical sensitivity of volatility decile portfolios to interest rate factor is not significant for 7 out of 10 portfolios. The three portfolios exhibiting significant interest rate exposure correspond to the two lowest volatility Deciles and to a high volatility Decile 9 portfolios. We confirm then Statement 3: although the interest factor itself has almost no explanation power, low volatility portfolios have small but positive interest rate sensitivity, similar to that of short-term (1.5 - 2 year maturity) bonds.

Our results shed more light on the relation between stock volatility and their sensitivity to interest rates. In particular, we



complement the literature by testing extensively the significance of interest rate exposure of volatility decile portfolios, across different factor model specifications and across different time sub-periods. We report evidence of a significant, albeit quite low in absolute terms, sensitivity of low volatility portfolios to interest rates, proxied by 10-year government bond rates. Yet, this relation is confirmed on two out of three sub-periods of the time period under study. Further directions of research include testing the link on interest rate sensitivity of volatility portfolios and their fundamental characteristics, such as sector composition, dividend yield and size. We also plan to extend the study on other geographic zones, in particular on European and Japan volatility portfolios. Finally, we plan to extend the study by separating the interest rates into two effects: inflation and real rate changes.

This paper is organized as follows. In section 2 we describe our dataset. In section 3 we introduce the multi-factor model for estimation of interest rate sensitivity. This is followed by empirical results and discussion in section 4. Then section 5 concludes.

## 2 Description of the investment universe

### 2.1 US equity data sample

For this empirical study of the US market we use returns of stocks belonging to the S&P 500 index from January 1990 to December 2014.<sup>2</sup> This index is a proxy for

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the large capitalization part the US equity market and serves as a base universe for a number of well-known low volatility indices, such as S&P 500 Low Volatility index and a number of proprietary and custom minimum variance indices calculated by Standard & Poor's<sup>3</sup>.

We take US equity factor times series (Market, Size, Value and Momentum) from the Kenneth French website. The size (SMB), value (HML) and Momentum (UMD) factors correspond to long-short portfolios that are long the stocks with low attributes (such as market capitalization for the size portfolio, book-to-market ratio for the value portfolio and past 12-month return for the momentum portfolio) and short the stocks with high attributes, built in a way that each portfolio neutralizes

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<sup>3</sup>For example, as of October 31st 2014 the US-listed PowerShares S&P Low Volatility ETF, based on S&P Low volatility index had USD 4.6 Bn of AUM, and the Europe-listed Ossiam US Minimum Variance ETF, the biggest minimum variance ETF based on the S&P universe, had more than USD 740m of AUM

the pairwise correlation with the others. See Fama and French (1993) and Carhart (1997) for further details on the construction and properties of these factors.

For the interest risk factor one has a choice among many variations of US Treasury bond yields: from the short term (2-year) to the mid term (5-year) or the long-term (10-year). Alternatively, one can opt for an aggregated bond index containing bonds of different maturities, such as the Barclays US Aggregate Treasury Index. We choose here the 10-year US Treasury bond yields as the interest rate risk proxy. Nevertheless, the conclusions we present in this paper are robust to the choice of the interest rate risk factor definition.<sup>4</sup>

## 2.2 Volatility decile portfolios

Over the period January, 31, 1990 to December, 31, 2014, starting from the set of stocks in the S&P 500 Index, we build ten portfolios based on stock past total volatility. Within each portfolio, stocks are equally-weighted so to avoid any capitalization bias. As so, Decile 1 contains stocks with the lowest volatility and Decile 10 contains stocks with the highest volatility. Portfolios are rebalanced monthly, each third Friday of the month. Volatility is computed over 500 business days, the computation is made 3 business days prior to each rebalancing<sup>5</sup>. Net dividends are reinvested in each portfolio following S&P methodology. Table 1 gives an overview of

portfolio characteristics over the 25 year-period.

While there is no clear pattern in the ex-post absolute returns, the volatility deciles have clear patterns of risk and of risk-adjusted performance (Figure 1). No surprise, the risk measures of the decile portfolios (total volatility, beta) exhibit a monotonous increasing pattern, with annual volatility ranging between 11.64% and 34.10%, and CAPM beta between 0.49 and 1.85. A similar pattern is present in the maximal drawdown measure: lowest-volatility decile exhibiting only 34.23% drawdown, contrary to the impressive 83.47% drawdown for the highest volatility stocks, while drawdowns are not strictly monotonous for intermediate deciles 4-7. Portfolio efficiency (measured by Sharpe ratio) decreases as the volatility increases. Contrary to the widespread belief, portfolios with lower volatility tend to have also higher returns. These results confirm the so-called low volatility anomaly documented in the literature (Haugen and Heins, 1975; Haugen and Baker, 1991; Ang et. al, 2006; Haugen and Baker, 2010). Finally, we observe a very clear pattern in the dividend yield: this measure monotonously decreases with increasing portfolio volatility.

## 3 Multi-factor model description

In order to estimate empirical interest rate sensitivity, we perform standard linear regression of volatility decile portfolios monthly returns over the Fama-French-Carhart (FFC) model completed with our

<sup>4</sup>More precisely, we use the Generic Unites States on-the-run government (annual) yield on 10-year maturity instruments (Bloomberg ticker *USGG10YR Index*).

<sup>5</sup>We apply a cut-off of maximum 10% of missing return observations over the estimation window for each stock. This excludes the stocks that only recently have been included in the S&P 500 universe.

	Annualized Return	Annualized Volatility	Max Drawdown	Sharpe ratio	CAPM Beta	CAPM Alpha	Dividend yield
Decile1	11.92%	11.64%	-34.23%	0.74	0.49	4.84%	2.71%
Decile2	12.78%	13.24%	-41.44%	0.71	0.74	3.73%	2.06%
Decile3	12.86%	13.60%	-44.56%	0.70	0.79	3.44%	1.79%
Decile4	11.95%	15.74%	-52.67%	0.55	0.92	1.54%	1.69%
Decile5	13.60%	16.30%	-49.19%	0.63	0.96	2.86%	1.51%
Decile6	11.70%	17.34%	-60.54%	0.48	1.03	0.41%	1.37%
Decile7	13.87%	18.24%	-53.35%	0.58	1.07	2.27%	1.17%
Decile8	13.05%	19.61%	-61.18%	0.49	1.19	0.59%	0.94%
Decile9	9.61%	24.74%	-67.12%	0.25	1.46	-4.95%	0.71%
Decile10	10.10%	34.10%	-83.47%	0.20	1.85	-7.47%	0.39%

Table 1: Performance statistics of volatility deciles. *Source S&P, Datastream. Data from Jan, 31, 1990 to Dec, 31, 2014. The Sharpe Ratio is computed with a risk-free rate over the period of 3.34%.*

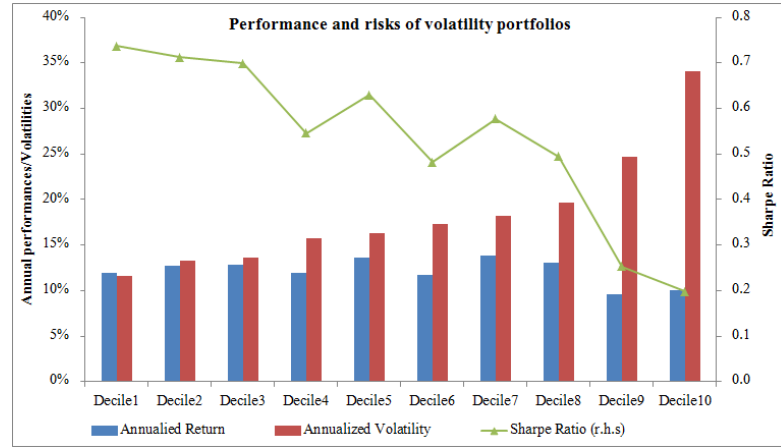


Figure 1: Performance statistics of volatility deciles. *Source S&P, Datastream. Data from Jan, 31, 1990 to Dec, 31, 2014. The Sharpe Ratio is computed with a risk-free rate over the period of 3.34%.*

proxy of the interest rate factor:

$$\begin{aligned}
 r_t^{Decile\ i} = & \alpha + \beta^{MKT} MKT_t \\
 & + \beta^{SMB} SMB_t + \beta^{HML} HML_t \\
 & + \beta^{UMD} UMD_t + \beta^{IR} IR_t + \epsilon_t
 \end{aligned} \quad (3.1)$$

The portfolio returns  $r^{Decile\ i}$  together with the FFC factors are measured each last business day of month.  $IR$  is the time series of monthly variations of 10-Year US Treas-

ury bond yields. Since we want to interpret the loading  $\beta^{IR}$  associated with this factor as an empirical equity duration, we multiply yield changes by -1:

$$IR_t = US10Y_{t-1} - US10Y_t;$$

Consequently positive loading means that portfolio return decreases when interest rates factor increases.



	MKT	SMB	HML	UMD
SMB	0.25***			
HML	-0.25***	-0.33***		
UMD	-0.25***	0.05	-0.14***	
IRT	-0.10*	-0.20***	0.04	0.19***

Table 2: Correlation matrix of factors in the regression model (3.1). *Significance: \*\*\* = 1% \*\* = 5% \* = 10%. No stars = correlation not significant*

Table 2 details correlations of the explanatory variables. The factors show significant correlations during the time period under study. In particular, the SMB and HML factors, designed to be weakly correlated, show significant negative correlation. Moreover, our proxy for the interest rate factor has significant negative correlation of -0.20 to the SMB factor, significant positive correlation of 0.19 to the UMD factor and significant negative correlation of -0.10 with the market factor. Thus, the model could potentially suffer from multicollinearity, which can result into artificially high loadings for some factors. We will address these issues in the following sections, testing the robustness of our results to different model specifications.

## 4 Empirical results

### 4.1 Explanatory power of the FFC model

We start by testing how good the Fama-French-Carhart (FFC) model is to explain the risk and the performances of the ten volatility portfolios over the past 25 years. Table 3 provides the results of a classic linear regression of monthly returns of volatility decile portfolios over the FFC risk factors: market return (MKT),

size (SMB), value (HML) and momentum (UMD), without adding the interest rate risk factor (i.e. model (3.1) with  $\beta^{IR} = 0$ ). The results give us some very interesting insights about these portfolios:

- Low volatility portfolios (deciles 1, 2 and 3) show positive and significant risk-adjusted returns. This is also true for deciles 5 and 7, albeit with weaker significance.
- The FFC model is able to capture the main variability drivers for intermediate (deciles 4 to 7) and high (decile 8 - 10) volatility portfolios -  $R^2$  are above 80%. The model has satisfactory explanation power for deciles 2 and 3, where the  $R^2$  is 73.35% and 79.07% respectively.
- The FFC model fails to explain the performances of decile 1 portfolio, where  $R^2$  is only 53%: almost half of the variability of this portfolio remains unexplained.
- Factor loadings exhibit several patterns. Market factor sensitivity is growing with volatility, that is expected. Low volatility portfolios, up to decile 4, have negative exposure to size factor, and the opposite is true for high-volatility deciles. Most of deciles have positive loadings for value factor,

Bucket	Alpha	MKT	SMB	HML	UMD	R <sup>2</sup>
Decile 1	0.004***	0.559***	-0.225***	0.433***	0.059**	53.27%
Decile 2	0.003***	0.783***	-0.221***	0.321***	-0.007	73.35%
Decile 3	0.003***	0.818***	-0.182***	0.351***	-0.049**	79.07%
Decile 4	0.002	0.958***	-0.116***	0.406***	-0.084***	82.89%
Decile 5	0.003**	0.984***	-0.02	0.425***	-0.115***	84.94%
Decile 6	0.001	1.06***	0.001	0.497***	-0.133***	88.55%
Decile 7	0.003**	1.079***	0.056	0.39***	-0.194***	87.64%
Decile 8	0.001	1.17***	0.157***	0.314***	-0.18***	89.40%
Decile 9	-0.001	1.364***	0.268***	0.103**	-0.352***	88.93%
Decile 10	0.001	1.613***	0.66***	-0.275***	-0.591***	85.83%

Table 3: 4 factor model regression of volatility decile portfolios. *Source S&P, Datastream, Kenneth French website. Data from Jan, 31, 1990 to Dec, 31, 2014. Significance: \*\*\* = 1% \*\* = 5% \* = 10%. No stars = loading not significant*

the exception being decile 10 with negative value exposure. Finally, all but one portfolio are negatively exposed to momentum factor, especially the highest volatility decile. The exception is decile 1 that has small positive momentum exposure.

#### 4.2 Interest rate exposure of volatility deciles

We run the regression of volatility portfolios' monthly returns over the augmented FCC model, including the interest rate risk factor. We remind that we flip the sign of the interest rates variation in order to interpret the loading of each portfolio as an empirical duration. We try to answer the following questions:

**Statement 1** Does the inclusion of the interest rate factor improve goodness-of-fit of the model?

**Statement 2** Does the inclusion of the interest rate factor eliminate or reduce the regression intercept (the alpha)?

**Statement 3** Is this empirical interest rate sensitivity statistically significant?

If the inclusion of the interest rate risk factor improves the goodness-of-fit of the model (measure by the  $R^2$ ), notably for low volatility portfolios, we can reasonably argue that interest rate is a relevant factor for explaining variability of these portfolios (Statement 1). Furthermore, should this inclusion make the portfolio's alpha smaller, we could conclude that a part of the unexplained risk-adjusted outperformance of low volatility portfolios is due to a hidden exposure to interest rates (Statement 2). Finally, in order to forecast low volatility performances in a future scenario with increasing interest rates, we need both a solid model that can explain the majority of the variability of these strategies and an accurate estimation of the empirical duration (Statement 3). The results are presented in Table 4.

First of all, we do not see significant increases in the goodness of the model's goodness-of-fit. For all deciles, the  $R^2$

Bucket	Alpha	MKT	SMB	HML	UMD	IR	R <sup>2</sup>
Decile 1	0.004***	0.559***	-0.194***	0.433***	0.039	1.824***	55.23%
Decile 2	0.003***	0.783***	-0.193***	0.321***	-0.025	1.598***	74.51%
Decile 3	0.003***	0.818***	-0.171***	0.351***	-0.057**	0.697*	79.28%
Decile 4	0.002	0.958***	-0.111***	0.406***	-0.088***	0.321	82.93%
Decile 5	0.003**	0.984***	-0.019	0.425***	-0.115***	0.03	84.94%
Decile 6	0.001	1.06***	0.004	0.497***	-0.135***	0.161	88.56%
Decile 7	0.003**	1.079***	0.05	0.39***	-0.19***	-0.323	87.67%
Decile 8	0.001	1.17***	0.148***	0.314***	-0.173***	-0.553	89.46%
Decile 9	-0.001	1.364***	0.243***	0.103**	-0.336***	-1.445***	89.20%
Decile 10	0.002	1.613***	0.654***	-0.275***	-0.587***	-0.369	85.84%

Table 4: 5 factor model regressions of volatility deciles. *Source S&P, Bloomberg, Datastream, Kenneth French website. Data from Jan, 31, 1990 to Dec, 31, 2014. Significance: \*\*\* = 1% \*\* = 5% \* = 10%. No stars = loading not significant*

improves only marginally when we add the interest rate factor. For decile 1 portfolio - the lowest volatility portfolio -  $R^2$  goes from 53.27% (Table 3) to 55.23% (Table 4). Moreover, no decile gets its alpha reduced or eliminated. In other words, the remaining unexplained alpha across volatility deciles is not explained by interest rate exposure. In this framework, Statements 1 and 2 are rejected. Only three out of ten deciles receive significant estimate of empirical duration: Deciles 1 and 2 have positive duration of approximately 1.824 and 1.598, while Decile 9 has negative duration of approximately -1.445. We find no significant duration for deciles 3 to 8 and decile 10. The coefficients of these deciles follow however a strict monotonic pattern: they decrease from a positive value to negative when volatility increases. This is similar to the inverse relation between volatility and interest rate duration found in Karyda et al. (2014). However, it is important to stress that the goodness-of-fit of the model is not significantly improved and the majority of the interest rates loadings

are not statistically significant.

According to these estimates and the fact the US rates fell from 8.5% in the early 1990s to 2.17% at the end of 2014, one should expect Decile 2 to outperform Decile 3 as they have very close FFC factor betas, and Decile 2 has twice the duration of Decile 3. Thus in a falling interest rate environment one could expect higher performance for Decile 2. Nevertheless, annualized realized performance over this period is 12.78% for Decile 2 against 12.86% for Decile 3 (Table 1). In other words, different interest rate exposure for these two deciles did not lead to noticeable performance difference.

#### 4.3 Sensitivity to factor model specification

In order to see whether the dependence among the explanatory factors could have produced artificially high loadings for interest rate factor we reproduce the same test with different specifications of the model (3.1). The simplest model

Bucket	Alpha	MKT	IRT	R <sup>2</sup>
Decile 1	0.006***	0.438***	2.424***	33.51%
Decile 2	0.004***	0.699***	1.98***	62.53%
Decile 3	0.004***	0.741***	0.936*	66.34%
Decile 4	0.002*	0.889***	0.344	71.67%
Decile 5	0.003**	0.935***	-0.225	74.45%
Decile 6	0.001	1.008***	-0.195	76.29%
Decile 7	0.003*	1.069***	-0.966*	78.52%
Decile 8	0.001	1.185***	-1.363***	83.98%
Decile 9	-0.002	1.475***	-2.991***	83.54%
Decile 10	-0.004	1.928***	-3.62***	74.76%

Table 5: 2 factor model regression of volatility deciles. *Source S&P, Bloomberg, Datastream, Kenneth French website. Data from Jan, 31, 1990 to Dec, 31, 2014. Significance: \*\*\* = 1% \*\* = 5% \* = 10%. No stars = loading not significant*

we can use is the one that includes only market factor and interest rate factor (i.e.  $\beta^{SMB} = \beta^{HML} = \beta^{UMD} = 0$ ).

We first remark that this model specification gives significantly lower  $R^2$  statistics. As in the FFC model extended with interest rate risk factor, we notice that intermediate volatility deciles (3 to 7) have no significant duration<sup>6</sup>. In this simple model high volatility Deciles 8-10 have significant negative duration with no significant alpha, and low volatility portfolios (Deciles 1 and 2) have significant positive duration and significant alpha. We note however that the interest rate duration coefficients are higher in absolute terms in the 2-factor model than in 5-factor model previously used. Finally, this simple model fails to fully capture the return variations of Decile 1: at least two third of the variation is left unexplained by the

model ( $R^2$  is 33.51%). In conclusion, this model gives some useful insights but is too poor to provide a fair description of low volatility portfolios.

We also test other specifications of the model, including progressively more factors in the regression and selecting to include or exclude the constant term. Tables 5 and 6 collects our findings for Decile 1, the portfolio composed of the lowest volatility stocks. Each row contains the interest rate factor loadings for a particular model version (3.1), where only some factors are included. For example, the first row displays the interest rate loading of Decile 1 portfolio in the model where  $\beta^{SMB} = \beta^{HML} = \beta^{UMD} = 0$ ; the second row represents the model where  $\beta^{HML} = \beta^{UMD} = 0$  and so on. The conclusions are in line with the already considered specifications for the factor model. Different model specifications have different levels of the goodness-of-fit, ranging from  $R^2 = 12.11\%$  (model without the market risk factor) to 52.06% (the

<sup>6</sup>Decile 3 and 7 do indeed have durations significant only at 90% level, but we exclude them as this level is generally rejected by statisticians

model with market, value and interest rate risk factor), but all have significant and positive interest rate loadings for this decile.

Summarizing at this point, we find that:

In the Fama-French-Carhart (FFC) framework, the inclusion of the interest rate risk does not significantly improve goodness-of-fit. The result holds true whatever the model specification we consider, and this is particularly true for low volatility portfolios.

The inclusion of interest rate risk factor has almost no impact on the risk-adjusted return generated by the portfolios (where the alpha is significantly different from zero).

It is possible to quantify the empirical duration of volatility portfolios with sufficient precision only for Deciles 1, 2 and 9.

#### 4.4 Interest rate exposure of volatility deciles: sub-periods

The estimations of empirical duration, as well as traditional Fama-French betas, are not static in time and might show significant variations. Table 7 collects factor beta estimations in three different sub-periods: January 1990 - June 1998, July 1998 - December 2006 and January 2007 - December 2014. Note that the model exhibits better goodness-of-fit over the latest period (2007/2014), with  $R^2$  statistics generally higher than the ones found Table 4. However even after this slight improvement, Decile 1 remains poorly explained by the model. The main result here is that empirical duration estimates change dramati-

cally over the three sub-periods, as one can see from Table 7 below.

**1990/1998** Decile 2 results in the most sensitive to interest rates (duration of 2.805). Surprisingly, it is the unique decile portfolio with a sufficiently significant duration: Decile 1 shows loading of 1.94 but with 10% chances of being non-significant.

**1998/2006** No decile portfolio shows significant interest rate sensitivity.

**2007/2014** Deciles 1 and 2 still have positive duration, but the error margin increased. We find Decile 3, 4 and 6 also with significant interest rate exposure. We note that ranking over the volatility does not coincide anymore with ranking over duration: Decile 4 has higher duration than 2 and 3, same for Decile 6 over 5.

Resuming, we see no time persistence in the results regarding interest rate sensitivity found in previous sub-sections. While the patterns concerning loadings to Fama-French-Carhart factors are similar, with increasing market beta, positive value exposure (except Decile 10) and negative momentum exposure, we do not find the same significance pattern for interest rate loading across sub-periods. The overall period results for low volatility portfolios (positive and significant interest rate sensitivity for Deciles 1 and 2) are mainly driven by the most recent sub-period 2007/2014. There are important changes in sensitivity for high volatility Deciles (8-10) across time. If in the overall regression it is Decile 9 that exhibits significant negative duration, we do not find this result on any of the sub-periods. The only high-volatility



Model	IRT	Alpha	R <sup>2</sup>
IRT - MKT	2.424***	0.006***	33.51%
IRT - MKT - SMB	1.772***	0.006***	41.67%
IRT - MKT - HML	2.357***	0.004***	52.06%
IRT - MKT - UMD	2.554***	0.006***	33.86%
IRT - SMB - HML - UMD	1.791***	0.01***	12.11%
IRT - MKT no intercept	2.621***	-	33.38%

Table 6: Interest rate loadings across various regression models for Decile 1. *Source S&P, Bloomberg, Datastream, Kenneth French website. Data from Jan, 31, 1990 to Dec, 31, 2014. Significance: \*\*\* = 1% \*\* = 5% \* = 10%. No stars = loading not significant*

decile with significant interest rate exposure appears to be Decile 8 in the sub-period 2007/2014. Note that for the majority of deciles, no matter the sub-period, the remaining alpha is hardly significant.

estimates of interest rate duration of portfolios with different levels of volatility, using a linear 5-factor model based on the Fama-French-Carhart factors (market, size, value and momentum) completed with the interest rate risk factor. Taking into account the potential issues of multicollinearity and time variation, we assess the robustness of interest rate sensitivity estimates with respect to changes in factor model specification and time period considered.

## 5 Conclusions

An interesting debate takes place around the possibility that low volatility equity investments are highly exposed to interest rate risk. This hidden risk is supposed to be responsible for the abnormal CAPM alpha of these strategies. Finally, in the context of future rises of interest rates, especially in the US, the advocates of this thesis argue that these strategies are set to under-perform.

Our paper addresses the question of interest rate sensitivity in wider perspective, looking not only at low volatility portfolios, but studying portfolios spanning the entire volatility spectrum. We provide empirical

We find significant and model-specification robust interest rate sensitivities for portfolios with low volatility (Deciles 1 and 2). This exposure is in the range 1.5 to 2, the exposure similar to that of short-term bonds. However this result was not confirmed on all of the considered sub-periods of the whole dataset. In particular, during 1998/2006 there was no significant interest rate exposure found for these portfolios.

Volatility portfolios with moderate volatility have exhibited no significant interest rate duration under all model specifications over the whole period of study, and occasionally showed significant duration during

Bucket	Alpha	MKT	SMB	HML	UMD	IRT	R <sup>2</sup>
January 1990 - June 1998							
Decile 1	0.004	0.635***	-0.225**	0.25**	0.12	1.964*	60.40%
Decile 2	0.005**	0.829***	-0.17**	0.023	-0.168**	2.805***	76.79%
Decile 3	0.007***	0.856***	-0.177***	0.001	-0.159***	0.852	82.95%
Decile 4	0.002	1.06***	-0.132**	0.061	-0.117*	-0.428	86.10%
Decile 5	0.002	1.091***	-0.033	0.191**	-0.232***	-0.274	84.83%
Decile 6	0.001	1.129***	-0.075	0.322***	-0.224***	-0.65	86.61%
Decile 7	0	1.206***	0.08	0.163**	-0.239***	-0.827	87.70%
Decile 8	0	1.363***	0.182**	0.355***	-0.185***	-0.847	87.48%
Decile 9	-0.001	1.321***	0.354***	0.049	-0.164**	-1.107	87.14%
Decile 10	-0.001	1.499***	0.672***	0.117	-0.302**	-1.783	75.92%
July 1998 - December 2006							
Decile 1	0.001	0.581***	-0.035	0.726***	-0.039	0.819	63.04%
Decile 2	0.002	0.818***	-0.113**	0.611***	-0.064*	0.963	76.68%
Decile 3	-0.001	0.88***	-0.053	0.71***	-0.107***	0.272	84.30%
Decile 4	0	0.963***	-0.076	0.691***	-0.103***	-0.486	83.97%
Decile 5	0.001	1.027***	0.015	0.698***	-0.113***	-0.384	84.74%
Decile 6	-0.001	1.018***	0.044	0.672***	-0.151***	0.147	86.63%
Decile 7	0.002	1.151***	0.095**	0.684***	-0.162***	-0.124	89.58%
Decile 8	0.002	1.166***	0.167***	0.433***	-0.166***	-0.498	87.84%
Decile 9	0	1.392***	0.227***	0.137	-0.358***	-1.505	87.57%
Decile 10	0.008*	1.764***	0.651***	-0.492***	-0.638***	1.601	88.78%
January 2007 - December 2014							
Decile 1	0.003	0.592***	-0.123	0.107	0.081*	2.081**	62.01%
Decile 2	0.001	0.79***	-0.008	-0.012	0.027	1.267**	89.25%
Decile 3	0.002	0.869***	0.012	-0.046	0.004	1.283***	92.33%
Decile 4	-0.001	1.013***	0.265***	-0.046	-0.083***	1.732***	94.43%
Decile 5	0.003*	1.005***	0.28***	0.006	-0.112***	0.558	94.95%
Decile 6	-0.002*	1.184***	0.26***	0.024	-0.136***	0.975**	96.53%
Decile 7	0.002*	1.095***	0.189**	-0.051	-0.253***	-0.679	93.67%
Decile 8	0	1.13***	0.253***	-0.008	-0.241***	-1.777***	94.89%
Decile 9	0	1.34***	0.326***	0.09	-0.331***	-1.364	92.19%
Decile 10	-0.001	1.516***	0.276**	0.254**	-0.543***	-0.481	92.39%

Table 7: 5 factor model regression of volatility deciles on sub-periods periods. *Source S&P, Bloomberg, Datastream, Kenneth French website. Data from Jan, 31, 1990 to Dec, 31, 2014. Significance: \*\*\* = 1% \*\* = 5% \* = 10%. No stars = loading not significant*

the most recent sub-period. Finally, high volatility portfolios showed quite mixed results. Over the total period and within the 5-factor model including Fama-French-Carhart factors along with interest rate factor, Decile 10 (stocks with the highest volatility) shows no significant interest rate exposure, while the adjacent Decile 9 has significant negative interest rate duration. However it appeared impossible to confirm this result by changing models specifications and time period. Thus the relation of high volatility stocks to interest rates is particularly difficult to measure.

The problems of instability of interest rate exposure estimates stem from two sources: multicollinearity in the factor set and a modest goodness-of-fit of the multi-factor model. Multicollinearity, which does not harm the overall goodness-of-fit, can lead to important errors in factor loadings, that can manifest themselves via an instability of the loadings to the choice of observation period. This seems to be particularly true for high-volatility portfolios (Deciles 8-10), as multi-factor model fails to give consistent duration estimates across time and across different model specifications.

We encounter the other kind of estimation problem in the case of low-volatility portfolios. Here the estimates of interest rate durations are quite robust to model specification and are confirmed across two out of three sub-periods under study. However, multi-factor models have an overall poor explanatory power for these portfolios, especially for Deciles 1 and 2. This means that, even if a significant small positive duration is confirmed for these portfolios, the addition of interest rate factor has not improved the model capability to capture variation sources for low-volatility

portfolios' returns, and thus the prediction that one can make with such a model would have very limited value.

Our study contributes to the emerging literature on the relation between portfolio volatility and its exposure to non-equity risks, in particular interest rate risk. We find similar pattern on interest rate exposure of volatility portfolios as Karyda et al. (2014), and provide an extensive robustness testing of this finding, that was not covered before in the literature. Our analysis shows that difference in risk-adjusted performance of low volatility and high volatility stocks cannot be explained by the addition of interest rate factor.

To go further, we will address in the future research the questions of the dependence of interest rate exposure of stocks from their fundamental characteristics, such as sector classification, size and dividend yield. We plan also to refine the factor model splitting the interest rate factor into two independent explanatory variables: real interest rates and inflation, as these two factors are not supposed to have the same relation to the company stock pricing.

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80, avenue de la Grande Armée  
75017 Paris France  
info@ossiam.com  
+ 33 1 78 40 56 90